Basic Idea

$$m^2, \lambda, \mu \longrightarrow \mathcal{L}_0 = \frac{Z}{2}(\partial\phi)^2 + \frac{m_0^2}{2} Z \phi^2 + \frac{\lambda_0}{4!} Z^2 \phi^4$$

$$m'^2, \lambda', \mu' \longrightarrow \mathcal{L}_0' = \frac{Z'}{2}(\partial\phi')^2 + \frac{m_0'^2}{2} Z' \phi'^2 + \frac{\lambda_0'}{4!} Z'^2 \phi'^4$$

Require $\left.\begin{aligned} \lambda_0 &= \lambda_0' \\ m_0 &= m_0' \end{aligned}\right\} \Rightarrow \sqrt{Z}\phi = \phi_0$ and $\sqrt{Z'}\phi' = \phi_0'$

identical correlators and can be identified

$$\Rightarrow \mathcal{L}_0 = \mathcal{L}_0'$$

$$\Rightarrow \quad \mathcal{L} + \mathcal{L}_{CT} = \mathcal{L}' + \mathcal{L}'_{CT} \equiv \mathcal{L}_0$$

$$\frac{1}{2}(\partial\phi)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4 \qquad\qquad \frac{1}{2}(\partial\phi')^2 + \frac{m'^2}{2}\phi'^2 + \frac{\lambda'}{4!}\phi'^4$$

$\Rightarrow$ the two construction simply correspond to different splittings of the same <u>bare</u> Lagrangian $\mathcal{L}_0$

- $\forall\, \mu \in \mathbb{R}^+$ I can perform such equivalent construction

$$\lambda\,,\; m\,,\; \phi \longrightarrow \lambda(\mu)\,,\; m(\mu)\,,\; \phi(\mu)$$

- μ-choice optimal to study physics at $E \sim \mu$ where $\log \frac{E}{\mu} \lesssim O(1)$

- <u>Interpretation</u>

$$\mathcal{L}_0 = \mathcal{L}_R(\mu) + \mathcal{L}_{CT}(\mu)$$

$\Downarrow$

compensates quantum fluctuations at $k \gtrsim \mu$

$$\mathcal{Z}_1 = \int \mathcal{D}\phi \; e^{-\int \mathcal{L}_0}$$

$$= \int \mathcal{D}\phi_{UV} \, \mathcal{D}\phi_{IR} \; e^{-\int \mathcal{L}_0} = \int \mathcal{D}\phi_{IR} \; e^{-\int \mathcal{L}_R(\mu) + \text{finite}}$$

$$\underset{k \gtrsim \mu}{\phantom{\mathcal{D}\phi_{UV}}} \qquad \underset{k \leq \mu}{\phantom{\mathcal{D}\phi_{IR}}}$$

$k \gtrsim \mu$ $\quad$ $k \leq \mu$

$$\phi_{IR}(k_1) \cdots \phi_{IR}(k_n)$$

$$\int D\phi \; \phi(k_1)\dots\phi(k_n)\, e^{-\mathcal{L}_0(\phi)}$$

$$k_i \to \sigma k_i \qquad k_i \sim \Lambda \overset{Ex}{\equiv} 1\,\mathrm{GeV}$$

$$\int D\phi \left(\phi(\sigma\Lambda)\dots\phi(\sigma\Lambda)\right) e^{-\mathcal{L}_0(\phi)}$$

$$\int D\phi(k \gtrsim \sigma\Lambda)\, D\phi(k \lesssim \sigma\Lambda)\, e^{-\mathcal{L}_0(\phi)}$$

$$= \int D\phi(k \lesssim \sigma\Lambda)\, \phi(\sigma\Lambda)\dots\phi(\sigma\Lambda)\, e^{-\mathcal{L}_{eff}(\phi(\sigma\Lambda),\,\sigma\Lambda)}$$

$$\lambda \longrightarrow \lambda(\sigma\theta)$$

$$\mathcal{L}_0 = \frac{1}{2}(\partial\phi_0)^2 - \frac{m_0^2}{2}\phi_0^2 - \frac{\lambda_0}{4!}\phi_0^4$$

- μ-dependence of $\lambda(\mu)$, $m^2(\mu)$, $\phi(\mu)$ implicitly determined by

$$\mu\frac{d}{d\mu}\lambda_0 = 0 \qquad \mu\frac{d}{d\mu}m_0^2 = 0 \qquad \mu\frac{d}{d\mu}\phi_0 = 0$$

$$\Downarrow$$

Renormalization Group Equations

Why R.G. ?

$$p \to e^{x} p \qquad -\infty < x < +\infty$$

IR UV

$$x \sim \ln \mu/_{1\,\text{GeV}}$$

$\ln \mu \to \infty$ UV

$\ln \mu \to -\infty$ IR

$$\bullet \quad \mu \frac{d}{d\mu} \lambda_0 = \mu \frac{d}{d\mu}\left[\lambda \mu^{\varepsilon}\left(1+\frac{a_1}{\varepsilon}+\frac{a_2}{\varepsilon^2}+\dots\right)\right]$$

$$\mu \frac{d}{d\mu}\lambda$$

$$\Downarrow$$

$$= \left(\hat{\beta}(\lambda)+\varepsilon\lambda\right)\left(1+\sum_n \frac{a_n}{\varepsilon^n}\right) + \lambda\hat{\beta}\left(\sum_n \frac{a'_n}{\varepsilon^n}\right) = 0$$

$$\Rightarrow \quad \hat{\beta} = - \frac{\varepsilon\lambda + \sum_n \frac{\lambda a_n}{\varepsilon^{n-1}}}{1+\sum_m \frac{(1+\lambda\partial_\lambda)a_m}{\varepsilon^m}}$$

$$= -\left\{\varepsilon\lambda - \lambda^2 a'_1 + \frac{\lambda^2}{\varepsilon}\left[-a'_2 + a'_1\left(a_1+\lambda a'_1\right)\right] + \dots\right\}$$

⚠ $\mu \to \mu'$, $\lambda(\mu) \to \lambda(\mu')$, ... $\equiv$ invariance of UV finite observables

$\Downarrow$

$$\hat{\beta}(\lambda) \equiv \text{finite}$$

$\Rightarrow$ all $\frac{1}{\varepsilon}$, $\frac{1}{\varepsilon^2}$, ... terms in $\hat{\beta}$ should vanish !!

Ex

$$-a_2' + a_1'(a_1 + \lambda a_1') = 0$$

$a_1 \to a_2$ fixed by diff. eq.

$$\theta_1 = c_1 \lambda + c_2 \lambda^2 + \cdots$$

$$\theta_1' = c_1 + 2 c_2 \lambda + \cdots$$

$$\theta_2' = (c_1 + 2c_2 \lambda)(c_1 \lambda + c_2 \lambda^2 + c_1 \lambda + 2 c_2 \lambda^2)$$

$$= c_1^2 \lambda + O(\lambda^2)$$

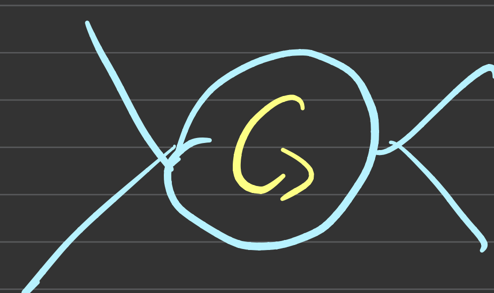
$\longrightarrow$ Known

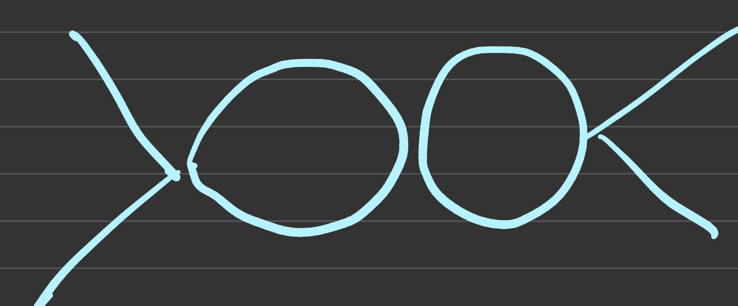
$$\boxed{\theta_2 = \frac{1}{2} c_1^2 \lambda^2 + O(\lambda^3)}$$

$$\mu \frac{d}{d\mu}\left[\underbrace{\lambda \mu^{\varepsilon}\left(1 + \frac{a_1}{\varepsilon} + \frac{a_2}{\varepsilon^2} + \cdots\right)}\right]$$

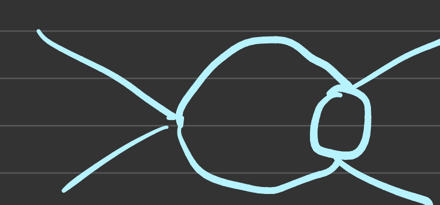
$$\lambda \mu^{\varepsilon}\left(1 + \frac{c_1 \lambda + \cdots}{\varepsilon} + \frac{\frac{1}{2} c_1 \lambda^2 + \cdots}{\varepsilon^2} \cdots\right)$$

$$\qquad\qquad\qquad \| \qquad\qquad\qquad\qquad \|$$





log Λ



<u>P.V.</u> $\frac{1}{\varepsilon} \rightarrow \ln \Lambda$

$$\Longrightarrow \quad \hat{\beta}(\lambda) = -\varepsilon\lambda + \lambda^2 a_1'$$

$$\equiv -\varepsilon\lambda + \beta(\lambda)$$

- For renormalized quantities $\varepsilon \to 0$ smooth

$$\Longrightarrow \quad \boxed{\mu \frac{d}{d\mu} \lambda(\mu) \overset{d=4}{=} \lambda^2 a_1'(\lambda) \equiv \beta(\lambda)}$$

- Similar story for $m^2(\mu)$:

$$m_0^2 = m^2\left[1 + \frac{b_1(\lambda)}{\varepsilon} + \frac{b_2(\lambda)}{\varepsilon^2} + \dots\right]$$

$$0 = \mu\frac{d}{d\mu} m_0^2 = \mu\frac{d}{d\mu} m^2\left[1+\sum_n \frac{b_n}{\varepsilon^n}\right] + m^2\left[\sum_n \frac{b_n'}{\varepsilon^n}\right]\left(-\varepsilon\lambda + \lambda^2 a_1'\right)$$

$$\Rightarrow \quad \mu\frac{d}{d\mu}\ln m^2(\mu) = -\frac{\sum_n \frac{b_n'}{\varepsilon^n}\left[-\varepsilon\lambda + \lambda^2 a_1'\right]}{\left[1+\sum_n \frac{b_n}{\varepsilon^n}\right]}$$

$$\equiv -\gamma_m(\lambda)$$

$$\phi_0 = \sqrt{Z}\,\phi(\mu)$$

$$Z = \left[1 + \frac{c_1(\lambda)}{\varepsilon} + \frac{c_2(\lambda)}{\varepsilon^2} + \dots\right]$$

$$0 = \mu \frac{d}{d\mu}\phi_0 \Rightarrow \mu \frac{d}{d\mu}\phi = -\gamma(\lambda)\,\phi$$

$$\gamma(\lambda) = \frac{1}{2}\,\mu\frac{d}{d\mu}\ln Z_1 = \frac{1}{2}\left(\frac{\partial}{\partial\lambda}\ln Z\right)\left(-\varepsilon\lambda + \lambda a_1'\right)$$

- Formal solutions of RG eqs.

- $$\frac{d\lambda}{d\ln\mu} = \beta(\lambda) \Longrightarrow \ln\frac{\mu'}{\mu} = \int_{\lambda(\mu)}^{\lambda(\mu')} \frac{d\lambda}{\beta(\lambda)}$$

$$d\ln\mu = \frac{d\lambda}{\beta(\lambda)}$$

- $$\frac{d\ln m^2}{d\ln\mu} = -\gamma_m(\lambda) \Longrightarrow \ln\frac{m^2(\mu')}{m^2(\mu)} = -\int_{\ln\mu}^{\ln\mu'} \gamma_m(\lambda(\tilde\mu))\, d\ln\tilde\mu$$

$$= -\int_{\lambda(\mu)}^{\lambda(\mu')} \frac{\gamma_m(\lambda)}{\beta(\lambda)}\, d\lambda$$

- $$\frac{d}{d\ln\mu}\phi(\mu) = -\gamma(\mu)\phi(\mu)$$

$$\Longrightarrow \mathcal{Z}(\mu',\mu) \equiv e^{-\int_{\lambda(\mu)}^{\lambda(\mu')} \frac{\gamma(\lambda)}{\beta(\lambda)} d\lambda}$$

$$\begin{cases} \dfrac{d}{d\ln\mu'}\mathcal{Z}(\mu',\mu) = -\gamma(\lambda(\mu'))\,\mathcal{Z}(\mu',\mu) \\ \mathcal{Z}(\mu,\mu) = 1 \end{cases}$$

$$\Longrightarrow \phi(\mu') = \mathcal{Z}(\mu',\mu)\,\phi(\mu) \Longrightarrow \frac{d\phi(\mu')}{d\ln\mu'} = -\gamma(\mu')\phi(\mu')$$

Examples

- $\lambda\phi^4$

  - $\lambda_0 = \lambda \mu^{\varepsilon}\left(1 + \frac{a_1}{\varepsilon} + \dots\right)$

  - $a_1 = \frac{3\lambda}{16\pi^2}\lambda + O(\lambda^2)$

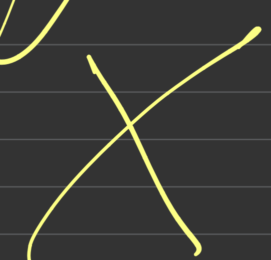
  - $\beta(\lambda) = \frac{3\lambda^2}{16\pi^2} + O(\lambda^3) > 0$

$$\mu \frac{d}{d\mu}\lambda(\mu) = \frac{3\lambda^2(\mu)}{16\pi^2} \Longrightarrow \boxed{\lambda(\mu') = \frac{\lambda(\mu)}{1 - \frac{3}{16\pi^2}\lambda(\mu)\ln\frac{\mu'}{\mu}}}$$

- $$\sigma(E) \approx \frac{1}{E^2} \cdot \lambda(E)^2 \left(1 + c\,\lambda(E) + \dots\right)$$

$$\approx \frac{1}{E^2}\,\lambda(E)^2 \approx \frac{1}{E^2}\left[\frac{\lambda(\mu)}{1 - \frac{3\lambda(E_0)}{16\pi^2}\ln E/\mu}\right]^2$$

$$\approx \frac{1}{E^2}\,\lambda(\mu)^2 \left(\sum \left(\frac{3\lambda}{16\pi^2}\right)^n L^n\right)$$

L^0  $+$ L^1  $+$ L^2 

L^2  $+ \dots$

$$L \equiv \ln\frac{E}{\mu}$$

$$\triangle \quad \frac{d\lambda}{d\ln\mu} = c_1\lambda^2 + c_2\lambda^3 + \cdots + c_\ell\lambda^{\ell+1} + \cdots$$

$$\frac{d\ln\lambda}{d\ln\mu} = \lambda\left(c_1 + c_2\lambda + \cdots + c_\ell\lambda^{\ell-1} + \cdots\right)$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ (\lambda L)^n & \lambda(\lambda L)^n & \lambda^{\ell-1}(\lambda L)^n \\ \lambda^n L^n & \lambda^{n+1} L^n & \lambda^{\ell+n-1} L^n \end{array}$$

⚠ RG asymptotics of $\lambda\phi^4$ in 4D

$$\lambda(\mu') = \frac{\lambda(\mu)}{1 - \frac{3}{16\pi^2}\lambda(\mu)\ \ln\frac{\mu'}{\mu}}$$

- IR $\quad \ln \mu'/\mu \longrightarrow -\infty \implies \lambda(\mu') \simeq \dfrac{16\pi^2}{3\ln\frac{\mu}{\mu'}} \longrightarrow 0$

- UV $\quad \dfrac{3\lambda(\mu)}{16\pi^2} \ln\dfrac{\Lambda_*}{\mu} = 1 \implies \lambda(\Lambda_*) = \infty \quad$ Landau Pole

$$\ln\frac{\Lambda_*}{\mu} = \frac{16\pi^2}{3\lambda(\mu)} \longrightarrow \Lambda_* = \mu\, e^{\frac{16\pi^2}{3\lambda(\mu)}}$$

⚠ Landau Pole & bare couplings

$\frac{1}{\varepsilon}$ in DR

- Pauli-Villars : $\lambda_0 = \lambda(\mu)\, P\left(\lambda(\mu), \ln\frac{\Lambda}{\mu}\right)$

$$\Downarrow$$

$$\sum_{n=0}^{\infty} p_n(\lambda)\left(\ln\frac{\Lambda}{\mu}\right)^n$$

$p_0 = 1$

$p_n = c\lambda^n + \dots$

- $\Lambda \simeq \mu$: $\lambda_0 = \lambda(\Lambda)\left[1 + a_1\lambda(\Lambda) + a_2\lambda^2(\Lambda) + \dots\right]$

hp weak coupling

$$\lambda_0 \sim \lambda(\Lambda) \simeq \frac{\lambda(\mu)}{1 - \frac{3}{16\pi^2}\lambda(\mu)\ln\frac{\Lambda}{\mu}}$$

$$\lambda_0 = \lambda(\mu) + \frac{3}{16\pi^2}\lambda(\mu)\ln\frac{\Lambda}{\mu} + \left(\frac{3\lambda(\mu)}{16\pi^2}\ln\Lambda/\mu\right)^2$$

$$+ \ldots$$

we must work under assumption

that $\frac{3\lambda(\mu)}{16\pi^2}\ln\Lambda/\mu \ll 1$

$$\Lambda \ll \Lambda_* \equiv \text{Landau pole scale}$$

$$(\partial\phi)^2 + \lambda\phi^4 + \frac{1}{\Lambda_*^2}\phi^6 + \cdots$$

Asymptotics of $g\phi^3$ in 6D

$$g_0 = g\mu^{\varepsilon/2}\left(1 - \frac{g^2}{64\pi^3}\frac{1}{\varepsilon} + \dots\right)$$

$$\mu\frac{d}{d\mu}g_0 = 0 \implies \hat{\beta}(g) = -\frac{\varepsilon}{2}g - \frac{3g^3}{128\pi^3}$$

- 6D $$\frac{dg}{d\ln\mu} = -\frac{3g^3}{128\pi^3} \implies \frac{dg^2}{d\ln\mu} = -\frac{3g^4}{64\pi^3}$$

$$g^2(\mu') = \frac{g^2(\mu)}{1 + \frac{3}{64\pi^3}g^2(\mu)\ln\frac{\mu'}{\mu}}$$

- UV : $\ln \frac{\mu'}{\mu} \to \infty \implies \tilde{g}^2(\mu') \sim \frac{64\pi^3}{3 \ln \frac{\mu'}{\mu}} \to 0$

<u>Asymptotic Freedom</u>

- IR $g(\mu_*) = \infty$ for $\frac{3}{64\pi^3} \tilde{g}^2(\mu) \ln \frac{\mu_*}{\mu} = -1$

$$\boxed{\mu_* = \mu\, e^{-\frac{64\pi^3}{3 g^2(\mu)}}}$$

⚠ Bare coupling (P. V.)

$$g_0^2 \simeq \frac{g^2(\mu)}{1+\frac{3}{64\pi^3}\, g^2(\mu)\ln\frac{\Lambda}{\mu}}$$

- $\left(g^2 \ln\frac{\Lambda}{\mu}\right)^n$ series of counter-terms makes sense also when $g^2\ln\frac{\Lambda}{\mu} \to \infty$

The Callan–Symanzik equation

$$\mathcal{O} \equiv \mathcal{O}\left(\lambda_a(\mu),\ \ln E/\mu\right)$$

- $\langle \phi_1(x_1) \cdots \phi_n(x_n) \rangle \longrightarrow (2\pi)^4 \delta\left(\sum p_i\right) \quad G(p_1, \ldots, p_n)$

$$\sqrt{Z_i}\,\phi_i = (\phi_0)_i \quad \Longrightarrow \quad G_0(p_1, \ldots, p_n) = \prod_i \sqrt{Z_i} \cdot G(p_1 \cdots p_n)$$

$$0 = \mu \frac{d}{d\mu} G_0 = \left(\mu \frac{d}{d\mu} \prod \sqrt{Z_i}\right) \cdot G + \left(\prod \sqrt{Z_i}\right) \mu \frac{d}{d\mu} G$$

$$0 = \left[\frac{1}{2} \sum_i \mu \frac{d}{d\mu} \ln Z_i + \mu \frac{d}{d\mu}\right] G$$

$$0 = \left[\sum_i \gamma_i + \mu \frac{d}{d\mu} \right] G$$

- $G = G(p_1, \ldots p_n, \lambda_a(\mu), \mu)$

$\lambda_a \equiv$ all parameters
$\equiv$ all couplings &
all masses

$$\boxed{\left(\sum_i \gamma_i + \sum_a \beta_a \frac{\partial}{\partial \lambda_a} + \mu \frac{\partial}{\partial \mu} \right) G = 0}$$

- **Ex** • $\frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4!} \phi^4$ $\qquad \lambda_0 \longrightarrow m^2, \lambda$

  • $G \equiv G(p_1 \cdots p_n, \lambda(\mu), m^2(\mu), \mu)$

$$\left(\mu \frac{\partial}{\partial \mu} + \beta(\lambda) \frac{\partial}{\partial \lambda} + \beta_{m^2} \frac{\partial}{\partial m^2} + n\gamma \right) G = 0$$

$$\Big(\underbrace{\mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial \lambda} - \gamma_m m^2 \frac{\partial}{\partial m^2}} + n\gamma \Big) G = 0$$

$$\left(\mu \frac{d}{d\mu} + n\gamma \right) G = 0$$

- <u>Formal solution</u>

$$Z(\mu,\tilde{\mu}) = e^{-\int_{\tilde{\mu}}^{\mu} \frac{d\mu'}{\mu'} \gamma(\lambda(\tilde{\mu}))} \qquad \mu \frac{d}{d\mu} Z(\mu,\tilde{\mu}) = -\gamma(\lambda(\mu)) Z$$

$$\mu \frac{d}{d\mu}\left(Z^{-n}(\mu,\tilde{\mu})\, G(P, \lambda(\mu), m^2(\mu), \mu) \right) = 0$$

$$\Rightarrow Z^{-n}(\mu,\tilde{\mu})\, G(P, \lambda(\mu), m^2(\mu), \mu) = G(P, \lambda(\tilde{\mu}), m^2(\tilde{\mu}), \tilde{\mu})$$

$$\boxed{G(P, \lambda(\mu), m^2(\mu), \mu) = Z^{n}(\mu,\tilde{\mu})\, G(P, \lambda(\tilde{\mu}), m^2(\tilde{\mu}), \tilde{\mu})}$$

- Scattering amplitudes

$$\langle \phi_0 \, \phi_0 \rangle = \frac{Z_0}{p^2 - m_R^2} + \cdots$$

$$\downarrow \text{RG inv}$$

$$\downarrow$$

$$Z_0 = \text{RG inv}$$

$$\mu \frac{d}{d\mu} Z_0 = 0$$

$\Rightarrow$ LSZ

$$\mathcal{M}(p_1 \cdots p_n) = \prod_i^n \frac{p_i^2 - m_R^2}{\sqrt{Z_0}} \langle \hat{\phi}_0(p_1) \cdots \hat{\phi}_0(p_n) \rangle = \text{RG inv}$$

$$\boxed{\mu \frac{d}{d\mu} \mathcal{M}(p_1 \cdots p_n) = 0}$$

- "Single scale" observables

$$\mathcal{O} = \mathcal{O}\left(E, \lambda(\mu), m^2(\mu), \mu\right) \qquad [\mathcal{O}] = \Delta$$

$$\equiv E^\Delta F\left(\frac{E}{\mu}, \frac{m^2(\mu)}{E^2}, \lambda(\mu)\right)$$

$$= E^\Delta F\left(1, \frac{m^2(E)}{E^2}, \lambda(E)\right)$$

Ex $2 \to 2$

$$= E^\Delta \lambda(E) \left[P_0\left(\frac{m^2(E)}{E^2}\right) + \lambda(E) P_1\left(\frac{m^2(E)}{E^2}\right) + \dots\right]$$

- systematic resummation of $\lambda^n \left(\lambda \operatorname{Log} \frac{E}{\mu}\right)^m$ contribution
- does not automatically capture possible

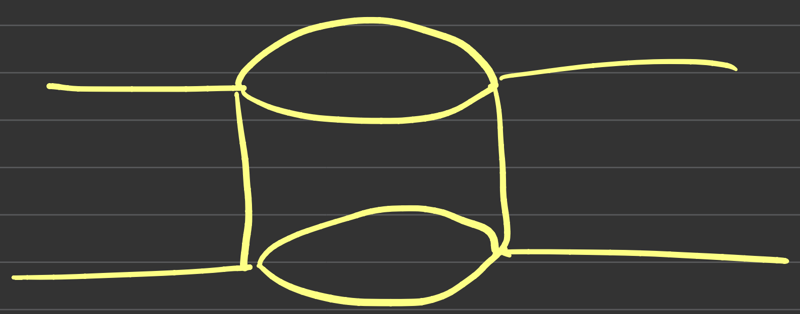
"IR divergences" $\sim \ln \frac{m^2}{E^2}$

Ex

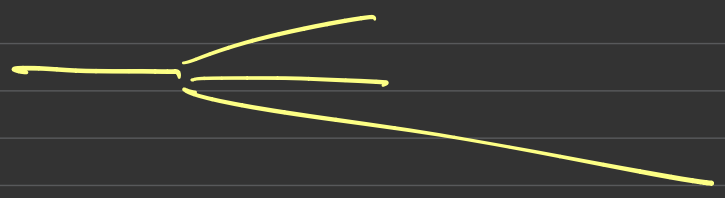
$$- m^2 \ln m^2/E^2 \longrightarrow$$

present in ϕ^4 but not in QED

$$- \lambda^4 \ln^2 \frac{m^2}{E^2} \longrightarrow$$



collinear divergences often present



• Ex of pure single scale

$e^+e^- \longrightarrow$ hadrons at all orders in α_s

$$\sigma(s) = \frac{\alpha_E^2(s)}{s} \sum_n^{\infty} c_n \, \alpha_s^n(s)$$

Fixed points

$\lambda \equiv \lambda^*$ consistently at all μ's

- If
  - $m = 0$
  - $\lambda_a = \lambda_a^* \mid \beta_a(\lambda_a^*) = 0$

$\xrightarrow{\text{c.s.}}$

$$\left(\mu \frac{\partial}{\partial \mu} + \sum_i n_i \gamma_i (\lambda^*)\right) G(p_1 \cdots p_n, \lambda^*, \mu)$$

- $$Z_1(\mu, \tilde{\mu}) = e^{-\int_{\tilde{\mu}}^{\mu} \gamma(\lambda^*)\, d\ln\mu'} = \left(\frac{\tilde{\mu}}{\mu}\right)^{\gamma_*} \qquad \gamma_* \equiv \gamma(\lambda_*)$$

$$\boxed{G\left(p,\, \lambda(\mu),\, m^2(\mu),\, \mu\right) = Z^{n}(\mu, \tilde{\mu})\, G\left(p,\, \lambda(\tilde{\mu}),\, m^2(\tilde{\mu}),\, \tilde{\mu}\right)}$$

• **Ex** $G_2(p,-p,\lambda^*,\mu) = Z^2(\mu,\tilde{\mu})\, G_2(p,-p,\lambda^*,\tilde{\mu})$

$$\tilde{\mu} \equiv \sqrt{p^2} \qquad = Z^2(\mu,\sqrt{p^2})\, G_2(p,-p,\lambda^*,\sqrt{p^2})$$

$$= \left(\frac{\sqrt{p^2}}{\mu}\right)^{2\gamma_*} \frac{1}{p^2}\left(c_0 + c_1\lambda^* + c_2\lambda^{*2} \cdots\right)$$

$$\propto p^{-2+2\gamma_*}$$

$\longrightarrow$ anomalus dimension

$$\Longrightarrow \langle\phi(x)\,\phi(0)\rangle \propto x^{-2-2\gamma_*}$$

as if $[\phi] = 1 + \gamma_*$

$\longrightarrow$ anomalous dimension